Kurs wyrównawczy - zajęcia zdalne z 16.12.2021

December 16, 2021

$$A^{-1}\cdot A = I$$

Lista 4.

a) $$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Mamy wzór $A^{-1} = \dfrac{[A_{ij}]^T}{\det A}$

$A_{ij} = (-1)^{i+j} M_{ij}$

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{5}{2} & -1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{3}{2} & 1 \end{bmatrix}$$

$$\det A = \left|\begin{matrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{matrix}\right| \begin{matrix} 1 & 2 \\ 1 & 0 \\ 2 & 1 \end{matrix} = 0 + 4 + \cancel{1} - 0 - \cancel{1} - 6 = -2 \neq 0$$

$$[A_{ij}]^T = \begin{bmatrix} +\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 1 \\ -5 & 1 & 3 \\ 2 & 0 & -2 \end{bmatrix}^T = \begin{bmatrix} 1 & -5 & 2 \\ -1 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix}$$

strona1.pdf

$\det A \neq 0$

Zad. 2

$$[A \mid I] \xrightarrow[\text{na wiersz.}]{\text{oper. el.}} [I \mid A^{-1}]$$

- dodawanie, odejm. wiersy
- mnożenie wierszy
- zamiane kolejności

a)
$$\left[\begin{array}{cccc|cccc} 1&0&0&1&1&0&0&0\\ 0&0&2&1&0&1&0&0\\ 0&1&1&1&0&0&1&0\\ 2&1&1&2&0&0&0&1 \end{array}\right] \xrightarrow{W_1\cdot(-2)+W_4=W_4'} \left[\begin{array}{cccc|cccc} 1&0&0&1&1&0&0&0\\ 0&0&2&1&0&1&0&0\\ 0&1&1&1&0&0&1&0\\ 0&1&1&0&-2&0&0&1 \end{array}\right] \xrightarrow{W_3'=W_3-W_4}$$

$$\left[\begin{array}{cccc|cccc} 1&0&0&1&1&0&0&0\\ 0&0&2&1&0&1&0&0\\ 0&0&0&1&2&0&1&-1\\ 0&1&1&0&-2&0&0&1 \end{array}\right] \xrightarrow[W_2'=W_2-W_3]{W_1'=W_1-W_3} \left[\begin{array}{cccc|cccc} 1&0&0&0&-1&0&-1&1\\ 0&0&2&0&-2&1&-1&1\\ 0&0&0&1&2&0&1&-1\\ 0&1&1&0&-2&0&0&1 \end{array}\right] :2 \rightsquigarrow \left[\begin{array}{cccc|cccc} 1&0&0&0&-1&0&-1&1\\ 0&0&1&0&-1&\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ 0&0&0&1&2&0&1&-1\\ 0&1&1&0&-2&0&0&1 \end{array}\right]$$

$$\xrightarrow{W_4'=W_4-W_2} \left[\begin{array}{cccc|cccc} 1&0&0&0&-1&0&-1&1\\ 0&0&1&0&-1&\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ 0&0&0&1&2&0&1&-1\\ 0&1&0&0&-1&-\frac{1}{2}&\frac{1}{2}&\frac{1}{2} \end{array}\right] \longrightarrow \left[\begin{array}{cccc|cccc} 1&0&0&0&-1&0&-1&1\\ 0&1&0&0&-1&-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&0&1&0&-1&\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ 0&0&0&1&2&0&1&-1 \end{array}\right]$$

strona2.pdf

strona3.pdf

$\det[3]=3$

Zad. 3. Rozwiązać równ. macierzowe

$$\begin{bmatrix}1 & 0\\ 3 & 3\end{bmatrix}\cdot X\cdot\begin{bmatrix}2 & 1\\ 3 & 5\end{bmatrix}=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$$

$A\cdot B\neq B\,A$

$\det A=\begin{vmatrix}1 & 0\\ 3 & 3\end{vmatrix}=3 \qquad \det B=\begin{vmatrix}2 & 1\\ 3 & 5\end{vmatrix}=7$

$$A\cdot X\cdot B=C \quad |_l\cdot A^{-1}$$
$$(A^{-1}\cdot A)\cdot X\cdot B=A^{-1}\cdot C$$
$$(I\cdot X)\cdot B=A^{-1}\cdot C$$
$$X\cdot B=A^{-1}\cdot C \quad |_p\cdot B^{-1}$$
$$X\cdot(B\cdot B^{-1})=A^{-1}\cdot C\cdot B^{-1}$$
$$X\cdot I=A^{-1}\cdot C\cdot B^{-1}$$
$$X=A^{-1}\cdot C\cdot B^{-1}$$

$$[A_{ij}]^T=\begin{bmatrix}3 & -3\\ 0 & 1\end{bmatrix}^T=\begin{bmatrix}3 & 0\\ -3 & 1\end{bmatrix}$$

$$A^{-1}=\frac{1}{3}\cdot[A_{ij}]^T=\begin{bmatrix}1 & 0\\ -1 & \frac{1}{3}\end{bmatrix}$$

$$[B_{ij}]^T=\begin{bmatrix}5 & -3\\ -1 & 2\end{bmatrix}^T=\begin{bmatrix}5 & -1\\ -3 & 2\end{bmatrix}$$

$$B^{-1}=\frac{1}{7}\cdot[B_{ij}]^T=\begin{bmatrix}\frac{5}{7} & -\frac{1}{7}\\ -\frac{3}{7} & \frac{2}{7}\end{bmatrix}$$

strona4.pdf

$$X = (A^{-1} \cdot C) \cdot B^{-1}$$

$A^{-1} \cdot C$

$$\begin{array}{cc|cc} & & 1 & 2 \\ & & 0 & 3 \\ \hline 1 & 0 & 1 & 2 \\ -1 & \frac{1}{3} & -1 & -1 \end{array}$$

$-2+1$

X

$$\begin{array}{cc|cc} & & \frac{5}{7} & -\frac{1}{7} \\ & & -\frac{3}{7} & \frac{2}{7} \\ \hline 1 & 2 & -\frac{1}{7} & \frac{3}{7} \\ -1 & -1 & -\frac{2}{7} & -\frac{1}{7} \end{array}$$

$\frac{5}{7} - \frac{6}{7}$ $\quad -\frac{1}{7} + \frac{4}{7}$

$-\frac{5}{7} + \frac{3}{7}$ $\quad \frac{1}{7} - \frac{2}{7}$

strona5.pdf

Lista 5 $\quad x=\frac{W_x}{W},\ y=\frac{W_y}{W},\ z=\frac{W_z}{W}$

c) 1) WZORY CRAMERA

$$\begin{cases} x+2y+3z=5 \\ x+3y+2z=1 \\ 3x+y+2z=11 \end{cases}$$

$$W=\left|\begin{matrix}1&2&3\\1&3&2\\3&1&2\end{matrix}\right|\begin{matrix}1&2\\1&3\\3&1\end{matrix}=\cancel{6}+12+3-27-\cancel{2}-\cancel{4}=12-24=-12\neq 0$$

$$W_x=\left|\begin{matrix}5&2&3\\1&3&2\\11&1&2\end{matrix}\right|\begin{matrix}5&2\\1&3\\11&1\end{matrix}=30+44+3-99-10-4=77-99-10-4=-22-14=-36$$

$$x=\frac{-36}{-12}=3$$

$$W_y=\left|\begin{matrix}1&5&3\\1&1&2\\3&11&2\end{matrix}\right|\begin{matrix}1&5\\1&1\\3&11\end{matrix}=2+30+33-9-22-10=-7+20+11=13+11=24$$

$$y=\frac{24}{-12}=-2$$

$$W_z=\left|\begin{matrix}1&2&5\\1&3&1\\3&1&11\end{matrix}\right|\begin{matrix}1&2\\1&3\\3&1\end{matrix}=33+6+5-45-1-22=11+11-46=22-46=-24$$

$$z=\frac{-24}{-12}=2$$

strona6.pdf

na skróty

$x = 3$

$y = -2$

z $\overline{\underline{II}}$ równania

$3+3(-2)+2z=1$

$3-6+2z=1$

$2z=4$

$z=2$

2) metoda Gaussa $\quad [A|B] \xrightarrow[\text{ne w.}]{\text{op. el.}} [E|X]$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 1 & 3 & 2 & 1 \\ 3 & 1 & 2 & 11 \end{array}\right] \xrightarrow[W_3' = W_3 - 3W_1]{W_2' = W_2 - W_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & -1 & -4 \\ 0 & -5 & -7 & -4 \end{array}\right] \xrightarrow[W_3' = W_3 + 5W_2]{W_1' = W_1 - 2W_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 13 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -12 & -24 \end{array}\right] :(-12)$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 5 & 13 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow[W_1' = W_1 - 5W_3]{W_2' = W_2 + W_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

strona7.pdf

$$(\alpha\cdot A)\cdot B=\alpha\cdot(A\cdot B)$$

3) z wykorzystaniem macierzy odwrotnej

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ 1 \\ 11 \end{bmatrix}$$

$$A\cdot X=B \quad \Big|\ ;A^{-1}$$

$$(A^{-1}\cdot A)\cdot X=A^{-1}\cdot B$$

$$I\cdot X=A^{-1}\cdot B$$

$$X=A^{-1}\cdot B$$

$$A^{-1}=-\frac{1}{12}\cdot\begin{bmatrix} -5 & -1 & -5 \\ 4 & -7 & 1 \\ -8 & 5 & 1 \end{bmatrix}$$

$$\det A=W=-12\neq 0$$

$$[A_{ij}]^T=\begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \end{bmatrix}^T=\begin{bmatrix} 4 & 4 & -8 \\ -1 & -7 & 5 \\ -5 & 1 & 1 \end{bmatrix}^T=\begin{bmatrix} 4 & -1 & -5 \\ 4 & -7 & 1 \\ -8 & 5 & 1 \end{bmatrix}$$

$$X=-\frac{1}{12}\cdot\begin{bmatrix} 4 & -1 & -5 \\ 4 & -7 & 1 \\ -8 & 5 & 1 \end{bmatrix}\cdot\begin{bmatrix} 5 \\ 1 \\ 11 \end{bmatrix}$$

strona8.pdf

$$
\begin{array}{c|c}
A^D \cdot B & \begin{matrix} 5 \\ 1 \\ 11 \end{matrix} \\
\hline
\begin{matrix} 4 & -1 & -5 \\ 4 & -7 & 1 \\ -8 & 5 & 1 \end{matrix} & \begin{matrix} -36 \\ 24 \\ -24 \end{matrix}
\end{array}
\qquad
X = -\frac{1}{12}\begin{bmatrix} -36 \\ 24 \\ -24 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}
$$

$\cancel{-25-1-55=-81 \quad -36}$ $\quad 20-1-55=-36$

$20-7+11=31-7=24$

$-40+5+11=-40+16=-24$

strona9.pdf

Zad. 2

$$\begin{cases} 6x_1 + 4x_3 = 2 \\ x_1 + a x_2 + x_3 = 3 \\ a x_1 - a x_2 + x_3 = -a \end{cases}$$

Kiedy ten układ jest Cramerowski.
Wtedy gdy $W \neq 0$

$$W = \begin{vmatrix} 6 & 0 & 4 \\ 1 & a & 1 \\ a & -a & 1 \end{vmatrix} \begin{matrix} 6 & 0 \\ 1 & a \\ a & -a \end{matrix} = \underline{6a} + 0 \underline{-4a} - 4a^2 + \underline{6a} - 0 = -4a^2 + 8a \neq 0 \, !$$

$$-4a^2 + 8a = 0$$

$$-4a(a-2) = 0$$

$$a = 0 \lor a = 2$$

1° $a \neq 0$ i $a \neq 2 \Rightarrow$ układ jest Cramerowski i ma dokładnie jedno rozwiązanie.

strona10.pdf

$2^\circ \quad a=0$

$$[A|B]=\left[\begin{array}{ccc|c} 6 & 0 & 4 & 2 \\ 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[]{\substack{W_1'=W_1-4W_3 \\ W_2'=W_2-W_3}} \left[\begin{array}{ccc|c} 6 & 0 & 0 & 2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{W_1'=W_1-6W_2}$$

$$\longrightarrow \left[\begin{array}{ccc|c} 0 & 0 & 0 & -16 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$0x_1+0x_2+0x_3=-16$

$0=-16$

układ jest sprzeczny

odp. $a\neq 0$ i $a\neq 2$ $\Rightarrow$ ukł. ozn.

$a=0 \Rightarrow$ ukł. spr.

$a=2 \Rightarrow$ ukł. nieoz.

$3^\circ \quad a=2$

$$[A|B]=\left[\begin{array}{ccc|c} 6 & 0 & 4 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & -2 & 1 & -2 \end{array}\right] \xrightarrow[W_2'=W_2-W_3]{W_1'=W_1-4W_3} \left[\begin{array}{ccc|c} -2 & 8 & 0 & 10 \\ -1 & 4 & 0 & 5 \\ 2 & -2 & 1 & -2 \end{array}\right] \xrightarrow[W_3'=2\cdot W_2+W_3]{W_1'=W_2(-2)+W_1} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 5 \\ 0 & 6 & 1 & 8 \end{array}\right]$$

układ jest nieoznaczony

strona11.pdf

Lista 6

Zad. 2

a) $\vec{u}=(p,-2,1)$ $\quad \vec{v}=(p,-p,1)$

Kiedy $\vec{u}\perp\vec{v}$? Wtedy gdy $\vec{u}\circ\vec{v}=0$

$$\vec{u}\circ\vec{v}=|\vec{u}||\vec{v}|\cos\sphericalangle(\vec{u},\vec{v})$$

$$\Downarrow$$

$$\cos\sphericalangle(\vec{u},\vec{v})=\frac{\vec{u}\circ\vec{v}}{|\vec{u}||\vec{v}|}$$

$$\cos 90^\circ=0$$

$$\vec{u}\circ\vec{v}=p^2+2p+1=0$$

$$(p+1)^2=0$$

$$p=-1$$

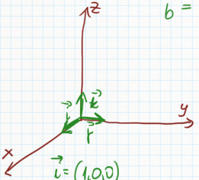
strona12.pdf

Zad. 3

$$\vec{a} = 2\vec{i} + \vec{j} - \vec{k} = 2(1,0,0) + (0,1,0) - (0,0,1) = (2,0,0) + (0,1,-1) = (2,1,-1)$$

$$\vec{b} = \vec{i} + \vec{j} + 2\vec{k}$$

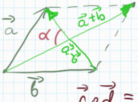
$$\vec{b} = (1,1,2)$$



$\vec{i} = (1,0,0)$

$\vec{j} = (0,1,0)$

$\vec{k} = (0,0,1)$



$$\vec{c} = \vec{a} + \vec{b} = (2,1,-1) + (1,1,2) = (3,2,1)$$

$$\vec{d} = \vec{a} - \vec{b} = (2,1,-1) - (1,1,2) = (1,0,-3)$$

$$\cos\alpha = \frac{\vec{c} \circ \vec{d}}{|\vec{c}| \cdot |\vec{d}|}$$

$$\vec{c} \circ \vec{d} = 3 \cdot 1 + 2 \cdot 0 + 1(-3) = 3 - 3 = 0$$

$$\vec{c} \perp \vec{d}$$

$$|\vec{a}| = \sqrt{4+1+1} = \sqrt{6}$$

$$|\vec{b}| = \sqrt{1+1+4} = \sqrt{6}$$

równoległobok jest rombem.

strona13.pdf

strona14.pdf

$\cos(\pi+\alpha) = -\cos\alpha$

zad.4 $\vec{a}\circ\vec{b} = ?$

$|\vec{a}| = 2$

$|\vec{b}| = 3$

$|\vec{a}\times\vec{b}| = 3$

$\sphericalangle(\vec{a},\vec{b}) \in \left(\frac{\pi}{2}, \pi\right)$

$\vec{a}\circ\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\underline{\cos\sphericalangle(\vec{a},\vec{b})}$

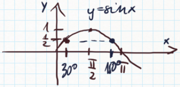
Skorzystamy ze wzoru

$|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\cdot\sin\sphericalangle(\vec{a},\vec{b})$

$3 = 2\cdot 3\cdot\sin\sphericalangle(\vec{a},\vec{b}) \quad |:6$

$\sin\sphericalangle(\vec{a},\vec{b}) = \frac{1}{2}$

$\sphericalangle(\vec{a},\vec{b}) = 150^\circ$



$\vec{a}\circ\vec{b} = 2\cdot 3\cdot\cos 150^\circ =$

$= 6\cdot\cos(180^\circ - 30^\circ) = 6\cdot(-(\cos(-30^\circ))) =$

$= 6\cdot(-\cos 30^\circ) = 6\cdot\left(-\frac{\sqrt{3}}{2}\right)$

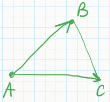
$= -3\sqrt{3}$

$\cos(-x) = \cos x$

strona15.pdf

Zad. 5

a) P_Δ $\quad A(-1,2,3) \quad B(4,1,2) \quad C(-1,4,1)$



$$P_\Delta = \frac{|\vec{AB}\times\vec{AC}|}{2}$$

$$\vec{AB} = (5,-1,-1)$$

$$\vec{AC} = (0,2,-2)$$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -1 & -1 \\ 0 & 2 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} - \vec{j}\begin{vmatrix} 5 & -1 \\ 0 & -2 \end{vmatrix} + \vec{k}\begin{vmatrix} 5 & -1 \\ 0 & 2 \end{vmatrix} = (4,10,10)$$

$$|\vec{AB}\times\vec{AC}| = \sqrt{4^2+10^2+10^2} = \sqrt{16+100+100} = \sqrt{216} = \sqrt{4\cdot 54} = 2\sqrt{54}$$

$$P_\Delta = \frac{2\sqrt{54}}{2} = \sqrt{54}$$

b) $P_{\square}$ $\qquad \vec{a} = (1,3,-4) \quad \vec{b} = (1,2,0)$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -4 \\ 1 & 2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 3 & -4 \\ 2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -4 \\ 1 & 0 \end{vmatrix} +$$

$$+\vec{k}\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = \vec{i}\cdot 8 - \vec{j}\cdot 4 + \vec{k}(-1) = [8,-4,-1]$$

$$P_{\square} = |\vec{a}\times\vec{b}| = \sqrt{8^2+(-4)^2+(-1)^2} = \sqrt{64+16+1} = \sqrt{81} = 9$$

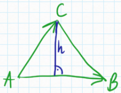
strona16.pdf

strona17.pdf

Zad. 6. $\triangle ABC$

$\vec{AB} = (1, 5, -3)$

$\vec{AC} = (-1, 0, 4)$



1) $P_\triangle = \frac{1}{2} P_{\square} = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

2) $P_\triangle = \frac{1}{2} |\vec{AB}| \cdot h$

$\Downarrow$

$$h = \frac{2P_\triangle}{|\vec{AB}|} = \frac{|\vec{AB} \times \vec{AC}|}{|\vec{AB}|}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & -3 \\ -1 & 0 & 4 \end{vmatrix} = \vec{i} \begin{vmatrix} 5 & -3 \\ 0 & 4 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -3 \\ -1 & 4 \end{vmatrix} +$$

$$+ \vec{k} \begin{vmatrix} 1 & 5 \\ -1 & 0 \end{vmatrix} = \vec{i} \cdot 20 - \vec{j} \cdot 1 + \vec{k} \cdot 5 = [20, -1, 5]$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{20^2 + (-1)^2 + 5^2} = \sqrt{400 + 26} = \sqrt{426}$$

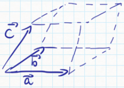
$$|\vec{AB}| = \sqrt{1^2 + 5^2 + (-3)^2} = \sqrt{1 + 25 + 9} = \sqrt{35}$$

$$h = \frac{\sqrt{426}}{\sqrt{35}} = \sqrt{\frac{426}{35}} \approx 3{,}5$$

strona18.pdf

Zad. 7 (wskazówka)

V_r : $\vec{a}=(1,3,-4)$ $\quad \vec{b}=(1,2,0)$ $\quad \vec{c}=(2,-1,-2)$



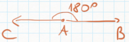
$$V_r=\left|\left(\vec{a}\,\vec{b}\,\vec{c}\right)\right|=\left|\left(\vec{a}\times\vec{b}\right)\circ\vec{c}\right|$$

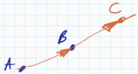
$$V_r=\left|\begin{vmatrix} 1 & 3 & -4 \\ 1 & 2 & 0 \\ 2 & -1 & -2 \end{vmatrix}\right| = \left|\cancel{-4}+\cancel{4}+0+16-0+6\right| = |22| = 22$$

$$\begin{matrix} 1 & 3 & -4 \\ 1 & 2 & 0 \end{matrix}$$

strona19.pdf

Zad. 9. $A(1,2,1)$ $B(2,4,1)$ $C(0,2,2)$





Czy A, B, C leżą na jednej prostej?

Np. wtedy gdy $\cos\sphericalangle(\vec{AB},\vec{AC}) = \pm 1$

albo $\sin\sphericalangle(\vec{AB},\vec{AC}) = 0$

albo $\exists_k \ \vec{AB} = k\cdot\vec{AC}$

metoda 1) $\cos\sphericalangle(\vec{AB},\vec{AC}) = \dfrac{\vec{AB}\circ\vec{AC}}{|\vec{AB}||\vec{AC}|}$

$\vec{AB} = (1,2,0)$

$\vec{AC} = (-1,0,1)$

$\vec{AB}\circ\vec{AC} = 1\cdot(-1)+2\cdot 0+0\cdot 1 = -1+0+0 = -1$

$|\vec{AB}| = \sqrt{1^2+2^2+0^2} = \sqrt{5}$

$|\vec{AC}| = \sqrt{(-1)^2+0^2+1^2} = \sqrt{2}$

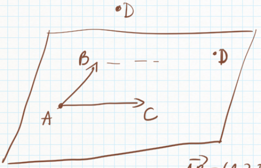
$\cos\sphericalangle(\vec{AB},\vec{AC}) = \dfrac{-1}{\sqrt{5}\cdot\sqrt{2}} = -\dfrac{1}{\sqrt{10}} \neq \pm 1$

Punkty nie leżą na jednej prostej.

strona20.pdf

$A(1,2,1)$ $\quad$ $D(1,4,2)$

Czy A, B, C, D leżą na jednej płaszczyźnie



Punkt D leży na tej samej płaszczyźnie co punkty A, B, C gdy

$V_r = 0$.

$$\vec{AB} = (1,2,0)$$
$$\vec{AC} = (-1,0,1)$$
$$\vec{AD} = (0,2,1)$$

$$\vec{AB}\,\vec{AC}\,\vec{AD} = \left|\begin{array}{ccc|cc} 1 & 2 & 0 & 1 & 2 \\ -1 & 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 0 & 2 \end{array}\right| = 0 + 0 + 0 - 0 - 2 + 2 = 0$$

odp. leżą na tej samej płaszczyźnie.